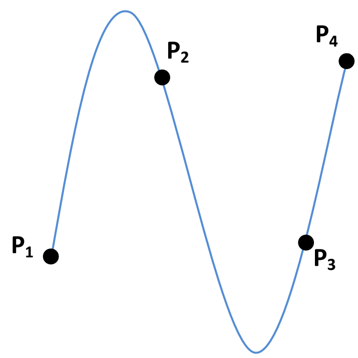
Polynomial Curve Fit

Polynomial curve fit determines the equation of an n[th] order polynomial that passes through n+1 points. Unlike polynomial *regression*, this method requires exactly n+1 data points and the resulting polynomial will pass through every point.



The figure above shows four points defining a cubic. The polynomial equation for a cubic is:

$$y = Ax^3 + Bx^2 + Cx + D$$

The variables A through D can be solved by a set of linear equations using the data from each of the points.

$$\begin{aligned}
Ax_1^3 + Bx_1^2 + Cx_1 + D &= y_1 \\
Ax_2^3 + Bx_2^2 + Cx_2 + D &= y_2 \\
Ax_3^3 + Bx_3^2 + Cx_3 + D &= y_3 \\
Ax_4^3 + Bx_4^2 + Cx_4 + D &= y_4
\end{aligned}$$

The equations are then converted to matrix form:

$$\begin{bmatrix} x_1^3 & x_1^2 & x_1 & 1 \\ x_2^3 & x_2^2 & x_2 & 1 \\ x_3^3 & x_3^2 & x_3 & 1 \\ x_4^3 & x_4^2 & x_4 & 1 \end{bmatrix} \cdot \begin{bmatrix} A \\ B \\ C \\ D \end{bmatrix} = \begin{bmatrix} y_1 \\ y_2 \\ y_3 \\ y_4 \end{bmatrix}$$

Solve for A through D:

$$\begin{bmatrix} A \\ B \\ C \\ D \end{bmatrix} = \begin{bmatrix} x_1^3 & x_1^2 & x_1 & 1 \\ x_2^3 & x_2^2 & x_2 & 1 \\ x_3^3 & x_3^2 & x_3 & 1 \\ x_4^3 & x_4^2 & x_4 & 1 \end{bmatrix}^{-1} \cdot \begin{bmatrix} y_1 \\ y_2 \\ y_3 \\ y_4 \end{bmatrix}$$