Scalar Triple Product

Basic Identity:

$$\begin{aligned}[A,B,C] &\equiv A \circ (B \times C) \\ &= B \circ (C \times A) \\ &= C \circ (A \times B) \\ &= \det(A\ B\ C) \\ &= \begin{vmatrix} A_1 & A_2 & A_3 \\ B_1 & B_2 & B_3 \\ C_1 & C_2 & C_3 \end{vmatrix}\end{aligned}$$

Additional Identities:

$$[A,B,C]D = [D,B,C]A + [A,D,C]B + [A,B,D]C$$

Vector Triple Product

Basic Identity:

$$A \times (B \times C) = B(A \circ C) - C(A \circ B)$$

Additional Identities:

$$\begin{aligned}(A \times B) \times C &= -C \times (A \times B) \\ &= -A(B \circ C) + B(A \circ C)\end{aligned}$$

Vector Quadruple Product

Basic Identity *(also known as Lagrange's Identity)*:

$$(A \times B) \circ (C \times D) = (A \circ C)(B \circ D) - (A \circ D)(B \circ C)$$

Additional identities :
Using $A^2 = A \circ A$ *and [A,B,C] as the scalar triple product* $A \circ (B \times C)$

$$\begin{aligned}(A \times B)^2 &= A^2B^2 - (A \circ B)^2 \\ A \times (B \times (C \times D) &= B\big(A \circ (C \times D)\big) - (A \circ B)(C \times D) \\ (A \times B) \times (C \times D) &= (C \times D) \times (B \times A) \\ &= [A,B,D]C - [A,B,C]D \\ &= [C,D,A]B - [C,D,B]A\end{aligned}$$