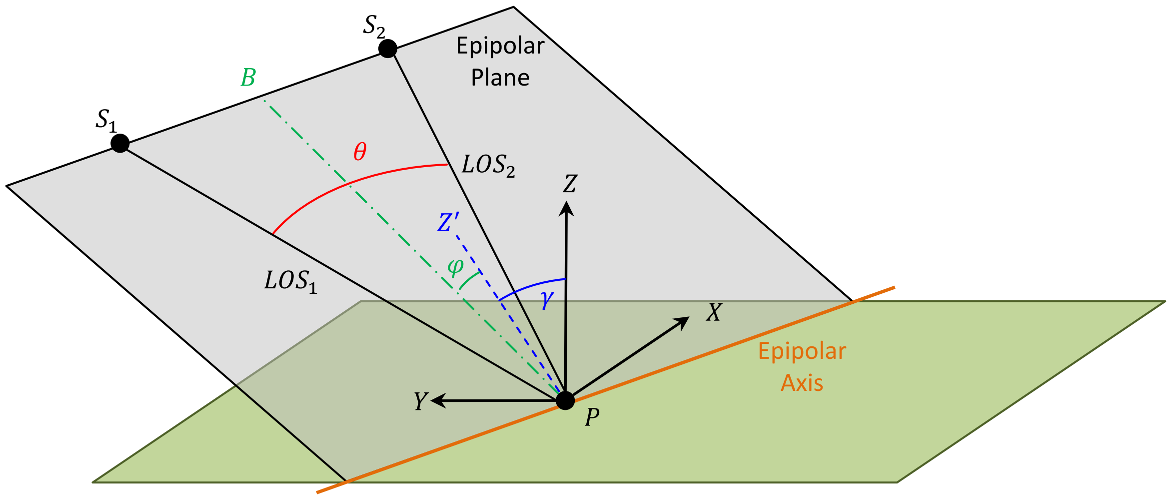
Epipolar Rectification

Epipolar rectification is the process of resampling an image pair such that they are able to be viewed in stereo. The process requires the calculation of the epipolar axis which defines the direction of movement between the cameras and the direction that the images must be aligned to be viewed in stereo. The geometry for epipolar alignment is shown below.

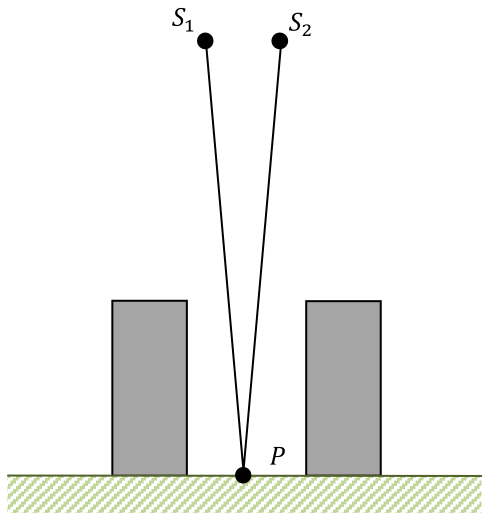


Where:

- S_1 and S_2 are the spatial locations of the imaging sensors
- P is the point of interest in both images
- θ is the convergence angle
- φ is the asymmetry angle
- γ is the bisector elevation angle
- B is the bisector vector
- Z' is the projection of the vertical axis (Z) on the epipolar plane.

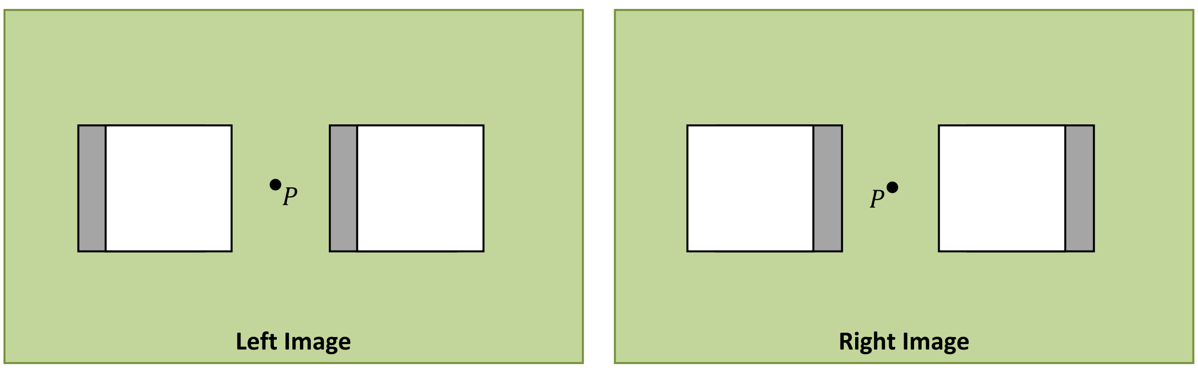
The epipolar plane is defined by the focal points of the two imaging systems and the stare point. As its name states, the bisector angle bisects the line of sight vectors of the two imaging systems. The location that the epipolar plane intersects the point of interest is the epipolar line. The projection of this line on to the images represents the horizontal axis of the epipolar pair. Rotating the images such that these axes are horizontal in both images will make the pair viewable in stereo.

There are three main properties that define the geometry of the pair: the convergence, asymmetry, and bisector elevation angles. An example scene is shown below that describes the effects of each of the stereo angles on the view. The rectification center point set on the ground between two buildings and the image collections are done from an airborne sensor at points S_1 and S_2.



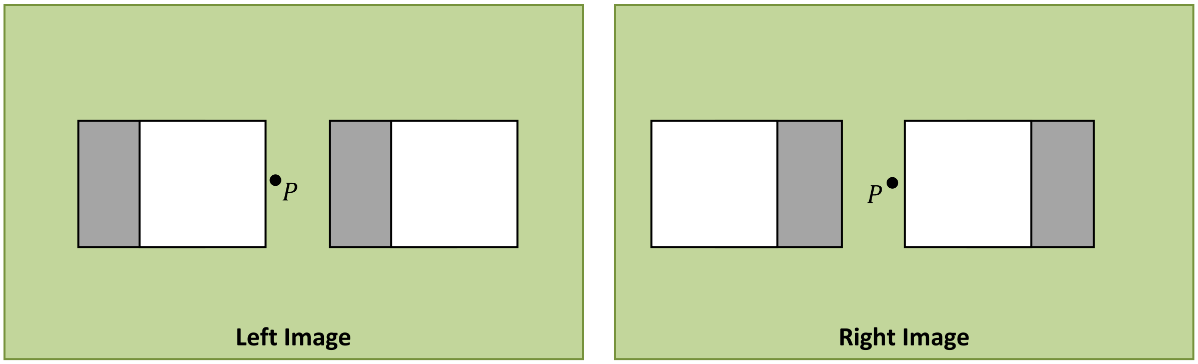
Spatial Scene

The convergence angle measures the angular separation between the two viewpoints. When the image pair is epipolar rectified, this value will determine the apparent relief of the pair. Starting with asymmetry and bisector elevation angles of 0°, the bisector vector is pointing directly nadir at the center point P. The stereo pair below is shown with these properties and a convergence angle of 10°



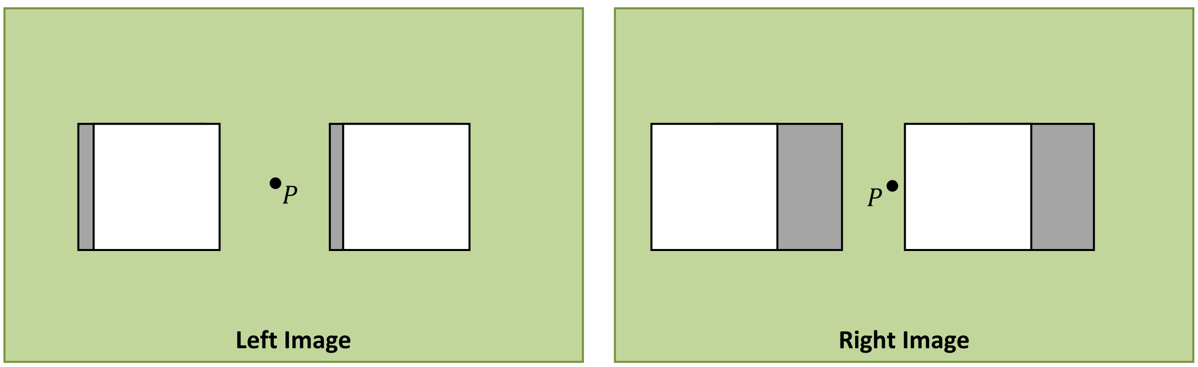
Convergence Angle (θ) *= 10°*
Asymmetry Angle (φ) *= 0°*
Bisector Elevation Angle (γ) *= 0°*

Expanding the convergence angle to 30° increases the apparent relief of the pair.



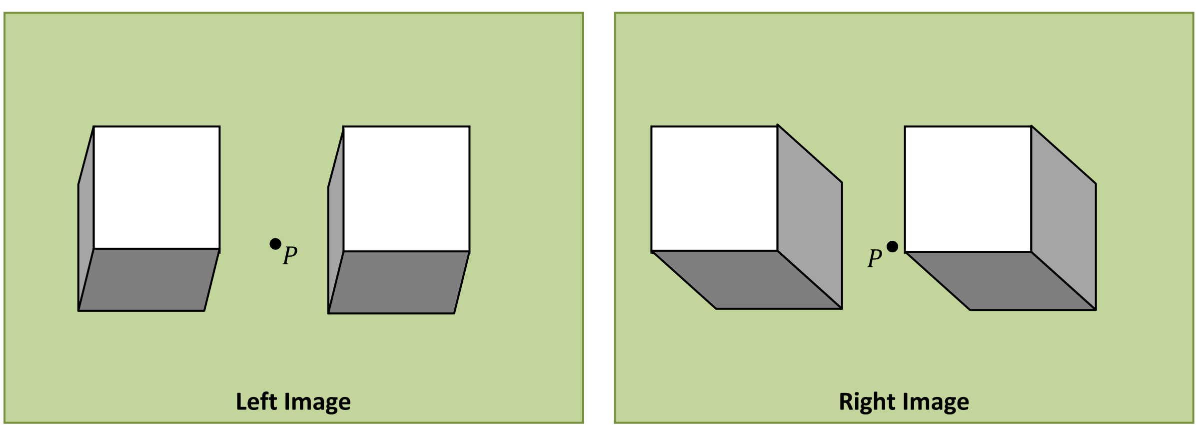
Convergence Angle (θ) *= 30°*
Asymmetry Angle (φ) *= 0°*
Bisector Elevation Angle (γ) *= 0°*

The asymmetry angle is the offset of the bisector vector from the center in the epipolar direction. If this angle is 0° the parallax in the left and right images is equivalent (in the epipolar direction). A non-zero value shifts the view in the epipolar direction to one image or the other.



Convergence Angle (θ) = 35°
Asymmetry Angle (φ) = 15°
Bisector Elevation Angle (γ) = 0°

The bisector elevation angle is the off-nadir angle in the direction perpendicular to the epipolar axis.



Convergence Angle (θ) = 35°
Asymmetry Angle (φ) = 15°
Bisector Elevation Angle (γ) = 30°

Epipolar Plane

To calculate the epipolar plane, the line of sight vectors of both platforms and the location of at least one of the sensors must be known. The normal vector of the plane can then be calculated by:

$$\hat{N} = \hat{L}_1 \times \hat{L}_2$$

Where $\hat{N}$ is the unit normal vector of the plane and $\hat{L}_1$ and $\hat{L}_2$ are the unitized line of sight vectors of the imaging platforms.

[TBD]